



On Optimal Method of Forecasting Inflation Rates Data Using Parsimonious GARCH Models

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Abstract - This paper considers the optimum method of forecasting inflation rates data of United States of America (developed economy) and Federal Republic of Nigeria (developing economy) using derived models. The models used were GARCH, Bilinear-GARCH (BL-GARCH), STAR-GARCH (EAR-GARCH, ESTAR-GARCH and LSTAR-GARCH) and ST-GARCH (ET-GARCH, EST-GARCH and LST-GARCH). From the results obtained from the analysis using E-view software, it was discovered that the hybrid of Bilinear with GARCH (BL-GARCH) out performed Classical GARCH model. In the same way the hybrids of STAR-GARCH and ST- GARCH also out performed better classical GARCH model. However, LSTAR-GARCH from STAR-GARCH and LST-GARCH from ST-GARCH out-performed other models (EAR-GARCH, ESTAR-GARCH, ET- GARCH and EST-GARCH). Conclusively, in this study, the optimum forecast model was produced by LSTAR-GARCH (STAR-GARCH).

Keywords: GARCH, BL-GARCH, STAR-GARCH, ST-GARCH, optimum method, inflation rates

1. Introduction

The schools of thought vary on the meaning and concept of inflation. However, the popular agreement among economists is that inflation is a continuous rise in the prices of goods and services. It could also be defined as a continuous soar in prices as measured by indicator such as the Consumer Price Index (CPI) or by the tacit price deflator for Gross National Product (GNP). The presence of inflation in an economy is best observed when currency loses purchasing power. Inflation rate forecasting or the concept of inflation rates volatility has been a discussion in so many literatures of economic and financial time series. Popular among them are Berument and Sahin (2010) who revealed volatility and forecasting of inflation is more important than its macro economists implications.

Model such as the Autoregressive Conditional Heteroscedasticity (ARCH) and her variants such as Generalised ARCH (GARCH) and Exponential GARCH (EGARCH) models were developed to model volatility fluctuation (non-constant) of such series. The ARCH model was initiative of Engle in (1982) this was further extended by Bollerslev in (1986) to a more generalized one called GARCH. The GARCH model has been used most commonly for the specification of the ARCH model. It imposes a kind of limitations on the parameters so as to assume positive variances. Nelson (1991) propounded an alternative to the GARCH model by modifying or extending the GARCH model to a more advanced Exponential GARCH (EGARCH) model. Unlike the GARCH, the EGARCH does not need imposition of restrictions on the parameters to assume a positive variance. Ling & Li (1997) and Coshall, J.T. (2008) made use of fractionally integrated moving Average (FIMA) models with conditional heteroscedasticity, by combining generalized autoregressive conditional heteroscedasticity (GARCH) and (ARIMA) models, their studies affirmed that financial data set exhibit conditional heteroscedasticity for this reason GARCH – type model are often used to model the inherent volatility.

Meyer et al; (1998) made use of autoregressive integrated moving average (ARIMA) to forecast Irish inflation and opined that ARIMA models are powerful relative to alternative (multivariate) model. Jean - Phillippe (2001) used the Box and Jenkins (1976) method to model and forecast Finnish inflation. Shittu, O. I., and Asemota. M. J. (2009) applied the like of Box and Jenkins (1976) method to forecast short term inflation in Croatia. In so many studies, many researches in the area of forecasting, also used the Box & Jenkins (1976) approach and tends to perform better in terms of forecasting in relation to other well-known time series models. Kwakye, J. K. (2004) studied the relationship between inflation and inflation uncertainty in the United Kingdom from 1973 to 2003 using both monthly and quarterly data they used different types of GARCH Mean (M)-Level (L) models which permits for instantaneous feedback between the conditional mean and variance of inflation data used to examine the relationship. He affirmed that there exist a positive relationship between past inflation and uncertainty about future inflation, in line with Frimpong, J. M., & Oteng-Abayie, E. F. (2006) whose their study of centered on testing for rate of dependence and asymmetric in inflation uncertainty, they opined that there was a relationship between inflation rate and inflation uncertainty. Alam, Z., & Rahman, A. (2012) looked at the forecast performance of different time series models for forecasting cocoa bean prices at Bagan Datoh cocoa bean. They used four different types of univariate time series models vis-a vis the exponential smoothing, autoregressive integrated moving average (ARIMA), generalized autoregressive conditional heteroscedasticity (GARCH) and the mixed ARIMA/ GARCH models.

The inflation rate data used in this study were that of United States of America and Federal Republic of Nigeria. They were obtained from the Consumer Price Index (CPI-U) known as U.S inflation rate data.com and that of Nigeria from Bureau of Statistics from 1996 to 2018 for both countries. The data covers period of 264 months. The choice of these two countries became imperative in order to confirm the reliability of the models used for both developed and developing economies.

2. General Representation

The final results of derivations for the variances of the models used for the computations in this paper as derived by Akintunde et.al 2013 Engle 1982 and Bollerslev 1986, are as shown below. The variance of GARCH model as an existing model was derived by Bollerslev as:

2.1 Generalized autoregressive model as derived is as follows:

$$Var(y_t) = \frac{\alpha_0}{1 - \sum_{i=1}^p (\alpha_i + \beta_j)} \quad 1$$

2.2 Bilinear-GARCH (BL-GARCH)

$$Var(y_t) = \frac{\alpha_0 + \sum \alpha_i \sigma_{t-1}^2 + \sum \beta_j \sigma_{t-1}^2}{1 - \tau_i^2} - \sigma_\varepsilon^4 \left(\sum \tau_i \right)^2 \quad \forall i \neq j \quad 2$$

2.3 STAR-GARCH models

$$\text{Var}\{y_{t(s-G)}\} = \frac{1}{1-\hat{\phi}_t^2} \left[\lambda_1^2 E(V_t^2) + \frac{\alpha_0}{1-\sum(\alpha_i + \beta_j)} \right] \quad 3$$

$$\phi_2' - \phi_1' = \lambda_1 \text{ and } V_t = y_{t-j} G_t \quad \forall j = 1, 2, \dots, p$$

2.4 ST-GARCH: The following models constituted ST-GARCH in this paper

(i) Exponential Transition-GARCH (ET-GARCH)

$$\text{Var}(G_{ET-GARCH}) = \gamma^2 \left\{ \frac{2\sigma^4}{n-1} \right\} \quad 4$$

(ii) Exponential Smooth Transition-GARCH (EST-GARCH)

$$\text{Var}\{G_{EST-GARCH}\} = 2\gamma^2 \sigma^2 \left[\frac{\sigma^2 + 2c^2 \sqrt{n-1} \sqrt{2}}{n-1} \right] \quad 5$$

(iii) Logistic Smooth Transition-GARCH (LST-GARCH)

$$\text{Var}\{G_{LST-GARCH}\} = 2\gamma^2 \sigma^2 \sqrt{\frac{2}{n-1}} \quad 6$$

3. Empirical Illustration

The software used for the empirical illustration is Econometric-view software popularly called E-view. And the following results were obtained for all the models used in the study.

3.1 GARCH MODEL

Based on Table1 below the estimated GARCH (1,1) model obtained for both U.S's and Nigeria's inflation rates are as follow:

$y_{U.S \text{ inflation rate}} = \sigma_t \varepsilon_t$ where σ_t and ε_t are obtainable from the fitted model:

$$y_{U.S \text{ interest rate}} = 0.9917 y_{t-1} + \varepsilon_t \text{ and } \sigma_t^2 = 0.4808 + 0.7598 \varepsilon_{t-1}^2 - 0.2539 (\sigma_{t-1}^2)$$

$y_{\text{Nigeria inflation rates}} = \sigma_t \varepsilon_t$ where σ_t and ε_t are obtainable from the fitted model:

$$y_{\text{Nigeria inflation rates}} = 1.0248 y_{t-1} + \varepsilon_t \text{ and } \sigma_t^2 = 2.0224 + 1.8368 \varepsilon_{t-1}^2 - 0.9965 (\sigma_{t-1}^2)$$

Table1: GARCH model fitted for the series

SERIES	COEFFICIENT (S.E)			MODEL VARIANCE
	α_0	α_1	β_1	
U.S INFLATION	0.4808 (0.0137)	-0.2539 (0.0854)	0.7598 (0.2300)	1.3214
NIG INFLATION	2.0224 (0.0526)	-0.9965 (0.0040)	1.8368 (0.0483)	407.1799

3.2. *Bilinear-Garch Models*

Estimation of parameters here was done in two stages, as the variances obtained from classical GARCH were used to obtain the parameters of Bilinear-GARCH models. The reason for the choice of bilinear (1.1) was due to the fact that few parameters make the models to be parsimonious; from where sets of data were generated and OLS applied and the following results were obtained for the series (U.S and Nigeria inflation rates). By using the values generated in table (2) the AGM fitted is

$$y_{tU.S \text{ inflation rate}} = \sigma_t \varepsilon_t + \frac{0.0032}{(0.0063)} y_{t-1} \varepsilon_{t-1} \quad \text{with variance of } 1.2562 \quad \text{and}$$

$$y_{tNig \text{ inflation rates}} = \sigma_t \varepsilon_t + \frac{0.6602}{(0.0016)} y_{t-1} \varepsilon_{t-1} \quad \text{with variance of } 57.7326.$$

Table 2: Bilinear-GARCH model fitted

SERIES	COEFFICIENT (S.E)	MODEL VARIANCE
U.S INFLATION	0.0032 (0.0063)	1.2562
NIG INFLATION	0.0660 (0.0016)	57.7326

3.3 *Smooth Transition Autoregressive Garch Models*

The initial values of γ and C were obtained using two dimensional grid searches. The smallest estimated values for the residual variance were selected. The two dimensional grid gave three possible values as tabulated in the tables (3) and (4). All the asterisk values were selected because they have minimum values.

Table 3: Values of grid of C

SERIES	I	II	III
U.S INFLATION RATES	0.35 •	155.76	30
NIGERIA INFLATION RATES	0.48 •	2.42	30

Table 1: Values of grid of γ

SERIES	I	II	III
U.S INFLATION RATES	0.50 [*]	10.00	30
NIGERIA INFLATION RATES	0.50 [*]	10.00	30

(i) EAR-GARCH

$$y_{t(S-G)} = \sigma_t \varepsilon_t + \phi_1' y_{t-1} \left(-\exp\left(\gamma(y_{t-1}^2)\right) \right) + \phi_2' y_{t-1} \left(1 - \exp\left(\gamma(y_{t-1}^2)\right) \right)$$

(i) $y_{U.S}$ inflation rate (S-G) = $\sigma_t \varepsilon_t + \underset{(0.1111)}{-2.3441} * y_{t-1} (1 - G_t) + \underset{(0.0083)}{0.1075} * y_{t-1} (G_t)$ and var. 1.1113

(ii) $y_{Nigeria}$ inflation rates (S-G) = $\sigma_t \varepsilon_t + \underset{0.9612}{-14.1042} * y_{t-1} (1 - G_t) + \underset{(0.011)}{0.5030} * y_{t-1} (G_t)$ and var. 95.3103

Table 5: Fitted model for EAR-GARCH series

SERIES	COEFFICIENT (SE)		Variance
	C(1)	C(2)	
U.S INFLATION	$\underset{(0.1111)}{-2.3441}$ $\underset{(0.0083)}{0.1075}$		1.1113
NIG INFLATION	$\underset{(0.96121)}{-14.1042}$ $\underset{(0.01113)}{0.50296}$		95.3103

(ii) ESTAR-GARCH

$$y_{t(S-G)} = \sigma_t \varepsilon_t + \phi_1' y_{t-1} \left(-\exp\left(\gamma(y_{t-1} - c)^2\right) \right) + \phi_2' y_{t-1} \left(1 - \exp\left(\gamma(y_{t-1} - c)^2\right) \right)$$

(i) $y_{U.S}$ inflation rates (S-G) = $\sigma_t \varepsilon_t + \underset{(0.0081)}{0.0988} * P_t + \underset{(0.0441)}{0.8726} * Q_t$ and var. 0.5735

(ii) $y_{Nigeria}$ inflation rates (S-G) = $\sigma_t \varepsilon_t + \underset{(0.00698)}{0.41596} * P_t + \underset{(0.35966)}{10.35029} * Q_t$ and var. 64.2819

Table 6: Fitted model for ESTAR-GARCH series

SERIES	COEFFICIENT (SE)		VARIANCE
	C(1)	C(2)	
U.S INFLATION	$\underset{(0.0081)}{0.0988}$ $\underset{(0.0441)}{0.8726}$		0.5735
NIG INFLATION	$\underset{(0.00698)}{0.41596}$ $\underset{(0.35966)}{10.35029}$		64.2819

(iii) LSTAR-GARCH

$$y_{t(S-G)} = \sigma_t \varepsilon_t + \phi_1' y_{t-1} \left(1 - \left(1 + \exp - (\gamma (y_{t-1} - c)) \right)^{-1} \right) + \phi_2' y_{t-1} \left(1 + \exp - (\gamma (y_{t-1} - c)) \right)^{-1}$$

(i) $y_{U.S \text{ Inflation rates}(S-G)} = \sigma_t \varepsilon_t + \underset{(0.0984)}{0.0803} * P_t + \underset{(0.0183)}{0.0240} * Q_t$ with var. 0.5047

(ii) $y_{Nigeria \text{ inflation rates}(S-G)} = \sigma_t \varepsilon_t - \underset{(0.39845)}{18.91187} * P_t + \underset{(0.00584)}{0.42033} * Q_t$ with var.57.9593

Table 7: Fitted model for LSTAR-GARCH series

SERIES	COEFFICIENT (SE)		VARIANCE
	C(1)	C(2)	
U.S INFLATION	0.0803 (0.0984)	0.0240 (0.0183)	0.5047
NIG INFLATION	-18.9119 (0.39845)	0.42033 (0.00584)	57.9593

3.4 Smooth transition GARCH model (ST- GARCH)

Using equations (4), (5) and (6), the variances of all the series for Smooth Transition GARCH models (ET-GARCH, EST-GARCH and LST-GARCH) were obtained and show that LST-GARCH has the minimum variance, followed by EST- GARCH and ET-GARCH in that order as shown in the table below. The implication of this is that LST-GARCH has the least variances and, as such, the best model.

Table 8: Computed Variances Of St-Garch

SERIES	ET-GARCH	EST-GARCH	LST-GARCH
US INFLATION RATES	0.0333	0.0132	0.0057
NIGERIA INFLATION RATES	244.2500	235.0794	15.3319

4. Empirical Comparison of Models

4.1. Variances Of Garch And Bilinear-Garch Models

Table 9. Summarized the results obtained for the variances of both classical GARCH models (GM) and Bilinear-GARCH models (AGM). From this table, the superiority of Bilinear-GARCH model was asserted on GARCH model as the variance of the GARCH model is greater than that of Bilinear-GARCH models. For instance, the variances of classical GARCH models for U.S's and Nigeria's inflation rates are 1.3214 and 407.1799 while Bilinear-GARCH models produces 1.2562 and 57.7326 respectively.

4.2. *Variances Of Bl-Garch With Garch Model*

Table 9: Variances Of Garch And Bilinear-Garch Models

SERIES	G.M	Bilinear-GARCH
U.S INFLATION RATE	1.3214	1.2562
NIG. INFLATION RATE	407.1799	57.7326

4.3. *Variances Of Star-Garch With Garch Model*

The table below shows the variances of all STAR-GARCH models with GARCH. It is clear from this table that the STAR-GARCH models out-performed the classical GARCH model. This is because the variances of all STAR-GARCH are minimal compared to classical GARCH model. However, LSTAR-GARCH appeared to be the best, followed by ESTAR-GARCH and EAR-GARCH in that order.

Table 10: Variances of STAR-GARCH with GARCH model

SERIES	GARCH MODEL	EAR MODEL	ESTAR MODEL	LSTAR MODEL
U.S INFLATION RATES	1.3214	1.1113	0.5735	0.5047
NIGERIA INFLATION RATES	407.1799	95.3103	64.2819	57.9593

4.4 *Variances Of Gm And All Smooth Transition Garch Model (St- Garch)*

Table (11) gives a clearer picture of the variances of classical GARCH model with ST-GARCH. It is evident that all ST-GARCH out-performed GARCH model. However, within this group, LST-GARCH has the least variances, followed by EST-GARCH and ET-GARCH in that order. If a researcher is considering forecasting with ST-GARCH, it is advisable to use LST-GARCH as it out-performed others.

Table 11: The Variances of ST-GARCH Models with Classical GARCH Model

SERIES	G.M	ET-GARCH	EST-GARCH	LST-GARCH
US INFLATION RATES	1.3214	0.0333	0.0132	0.0057
NIGERIA INFLATION RATES	407.1799	244.250	235.0794	15.3319

5. Conclusion

The variances of Bilinear-GARCH STAR-GARCH and ST-GARCH models are measure of improvement over GARCH model. However, LSTAR-GARCH model appeared to have produced the best model in this study and for the series used. This is closely followed by LST-GARCH and Bilinear-GARCH models in that order. The policy statement here is that for would be policy formulator/ analyst the use of LSTAR-GARCH is recommended. However, policy makers, investors can make use of other hybrids.

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