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Solar Radiation And Internal Heat Generation Effects On Unsaturated And Saturated Sandy Soil With Suction Velocity

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Abstract - The effects of solar radiation and internal heat generation on unsaturated and saturated sandy soil with suction velocity were investigated. The dimensional governing energy equation was reduced to non-dimensional form with the use of some dimensionless quantities, resulting into a partial differential equation of order two. The equation was further reduced to ordinary differential equation by perturbation method and then solved analytically. The effects of the solar radiation parameter, internal heat generation parameter and the Prandtl number were examined on the sandy soil both at unsaturated state and when it is saturated with water. The numerical results of these physical parameters computed by the use of Matlab R2009b were displayed on graphs for better illustrations. The increasing solar radiation and internal heat increased the temperature of the sandy soil at both states, while the Prandtl number decreased the temperature at the boundary layer. These outcomes however follow the results of existing literatures along the line of study.

Keywords: Solar radiation, Internal heat generation, unsaturated and saturated sand, suction velocity, perturbation method, Prandtl number.

1. Introduction

The study of heat transfer is a very important one because it has many areas of applications which cut across various fields of science, engineering and technology. Moreover, due to the importance of the temperature of the soil which determines the availability of basic nutrients useful for the growth of various plants, researchers have been attracted to the measurement of the warmth and coldness of these soils. For example, Onwuka [1] studied the effects of soil temperature on some soil properties and plant growth. He submitted that increase in soil temperature improves root growth while the tissue nutrient is reduced which in turn reduced the growth of plants' roots when the soil temperature is low. He added that soil temperature is a catalyst for many biological processes. Akinpelu, Olaleye and Adewoye [2] considered the effects of some physical parameters on ground temperature with time-dependent suction velocity in the presence of internal heat generation. Alam *et al* [3] studied the underground soil and thermal conductivity materials based heat reduction for energy-efficient building in tropical environment. Akinpelu, Alabison and Olaleye [4] worked on the variations in ground temperature in the presence of radiative heat flux and spatial dependent soil thermophysical property. Barbara *et al* [5] researched on heat conduction in the ground under natural conditions and with heat exchanger installed. In view of these literatures and host of others, this study focused of the effects of solar radiation and internal heat generation on unsaturated and saturated sandy soil with suction velocity and time-dependent thermal conductivity.

2. Mathematical Analysis

An unsteady two dimensional heat transfer through a porous medium with Dirichlet boundary condition is considered. The flow is infinite along horizontal axis ($y' - axis$) and $z' - axis$ which is inside the soil is taken to be normal to it. Since the flow is of extreme size along the horizontal axis, the flow field becomes the function of z' and t' alone [6]. The soil is considered to be an optically thin environment and the solar radiation is directly towards it in a direction along the gravity. Under the above assumptions, the governing equations under the usual Boussinesq's approximation became:

Continuity equation

$$\frac{\partial w'}{\partial z'} = 0 \tag{1}$$

Energy equation

$$\rho C_p\left(\frac{\partial T'}{\partial t'} + w'\frac{\partial T'}{\partial z'}\right) = k\frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z} + Q_0(T' - T'_\infty) \tag{2}$$

Subject to:

$$T' = T'_w \text{ at } z = 0 \tag{3}$$

$$T' \to T'_\infty \text{ as } z \to \infty \tag{4}$$

where, z' being the dimensional depth of the soil is perpendicular to y'. t' and w' are the time and the suction velocity respectively. T', T'_w and T'_∞ are the temperature, the wall temperature and the free stream temperature respectively. ρ, C_p, k and q'_r are density, specific heat capacity, thermal conductivity and the Radiative heat flux respectively.

Using the dimensionless parameters as used by Mohammed [6] and Akinpelu, Alabison and Olaleye [4],

$$t = \frac{t' w_0^2}{w},\ z = \frac{w_0 z'}{w},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ \omega = \frac{w\omega'}{w_0^2} \tag{5}$$

The suction velocity as used by Nwaigwe [7] is chosen to be time-dependent and given as

$$w' = -w_0(1 + \varepsilon A e^{i\omega' t'}) \tag{6}$$

where w_0, A and ω are the initial suction velocity, the suction parameter and the frequency of oscillation respectively. The negative sign signifies that the suction is towards the surface of the ground. A and ε are very small such that $\varepsilon A << 1$.

In addition, by Krishna and Reddy [8] the heat flux is given as

$$\frac{\partial q'_r}{\partial z'} = 4\alpha^2(T' - T'_\infty) \tag{7}$$

α being the absorption coefficient.

Also, Kareem and Salawu [9] and Akinpelu, Alabison and Olaleye [4], the time-dependent thermal conductivity is given as

$$k = k_0(1 + st) \tag{8}$$

s, t and k_0 are the variable thermal conductivity parameter, time and the constant thermal conductivity.

Equation (2) then becomes:

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t})\frac{\partial \theta}{\partial z} = \frac{1}{P_r}\left\{\frac{\partial}{\partial z}\left((1 + st)\frac{\partial \theta}{\partial z}\right)\right\} - R^2 + Q\theta \tag{9}$$

Subject to:

$$\theta = 1 \text{ at } z = 0 \text{ (the ground surface)} \tag{10}$$

$$\theta \to 0 \text{ as } z \to \infty \tag{11}$$

where,

$P_r = \frac{w\rho C_p}{k_0}$ (the Prandtl number)

$R^2 = \frac{4\alpha^2 \theta w}{w_0^2}$ (the radiation parameter), and

$Q = \frac{Q_0 w}{\rho C_p w_0^2}$ (the internal heat generation parameter)

3. Method of Solution

Equation (9) is a partial differential equation of second order which can be reduce to ordinary differential equation and solved analytically. In view of this, the regular perturbation method is employed and the assumed solution for the temperature gradient is given as:

$$\theta(z,t) = \theta_0(z) + \varepsilon e^{i\omega t}\theta_1(z) \quad (12)$$

Substituting equation (12) and its derivatives into (9), and neglecting the higher order terms $o(\varepsilon)^2$ with further simplifications, we obtain

$$(1+st)\theta_0'' + P_r\theta_0' + P_r Q\theta_0 = P_r R^2 \quad (13)$$

$$(1+st)\theta_1'' + P_r\theta_1' + \left(P_r Q - P_r i\omega\right)\theta_1 = -P_r A\theta_0 \quad (14)$$

where the primes represent ordinary differentiation with respect to z.
Using the assumed solution (12), corresponding boundary conditions (10) and (11) can be rewritten as follows:

$$\theta_0 = 1, \quad \theta_1 = 0 \quad \text{on} \quad z = 0 \quad (15)$$

$$\theta_0 \to 0, \quad \theta_1 \to 0 \quad \text{as} \quad z \to \infty \quad (16)$$

Solving equations (13) – (14) subject to the boundary conditions (15) – (16), the resulted transient temperature distribution is:

$$\theta_0 = C_1 e^{m_1 z} + C_2 e^{m_2 z} + C_3 \quad (17)$$

$$\theta_1 = C_4 e^{m_3 z} + C_5 e^{m_4 z} + C_6 e^{m_1 z} + C_7 e^{m_2 z} \quad (18)$$

With the assumed solution, the ground temperature becomes,

$$\theta = C_1 e^{m_1 z} + C_2 e^{m_2 z} + C_3 + \varepsilon e^{i\omega t}\left(C_4 e^{m_3 z} + C_5 e^{m_4 z} + C_6 e^{m_1 z} + C_7 e^{m_2 z}\right) \quad (19)$$

4. Results and Discussion

The numerical results of the transient temperature was computed and displayed on graphs. The Solar radiation parameter, the internal heat generation (Q) and the Prandtl number (Pr) were examined on the Temperature gradient of both the unsaturated and saturated sandy. Table 1 below shows the values of the thermal conductivities used for both the unsaturated and saturated sandy soils.

Table 1: Thermal conductivities of unsaturated and saturated sandy soils

Sandy soil	Thermal Conductivity (Btu/ft hr ^{0}F)
Unsaturated	0.44
Saturated	1.44

Gary [10], Wayne [11]

Moreover the following default values were adopted except otherwise stated.

$$P_r = 0.71,\ Q = 0.01,\ \omega = \frac{\pi}{2},\ \varepsilon = 0.01,\ t = 1.0,\ A = 0.5,\ R = 0.1,\ A_0 = 1$$

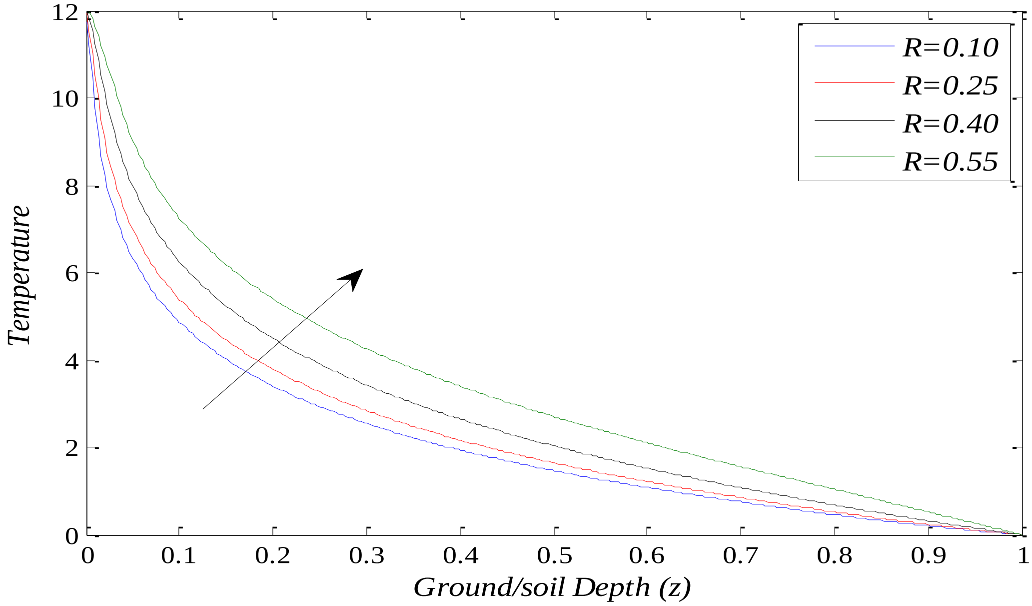

Figure 1: Temperature profile for various values of Radiation parameter for **unsaturated** sandy soil.

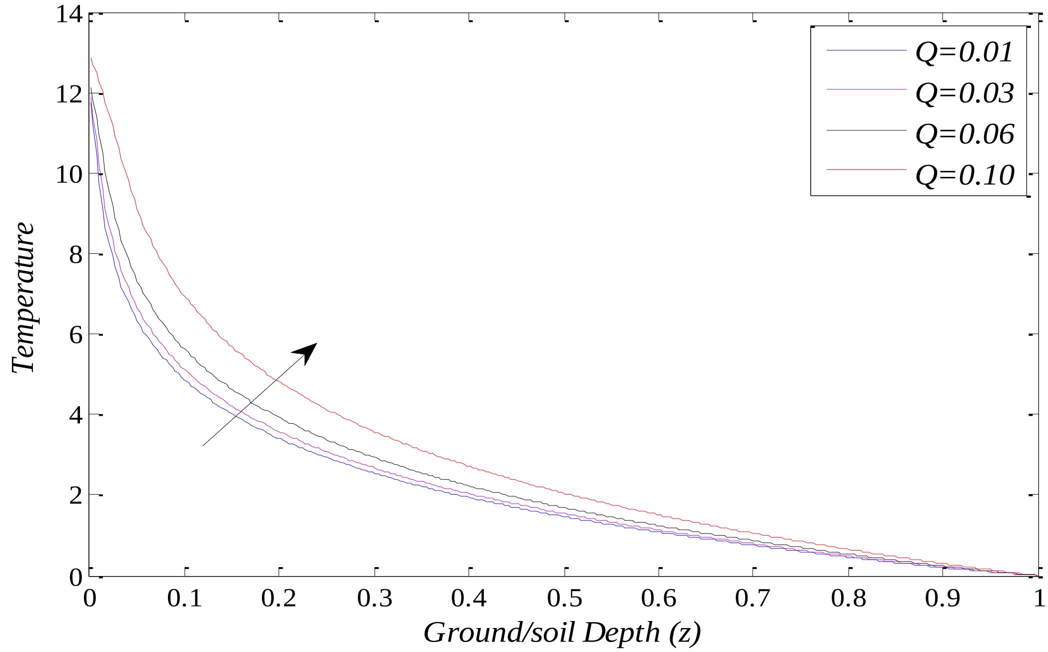

Figure 2: Temperature profile for various values of Internal heat generation parameter for **unsaturated** sandy soil

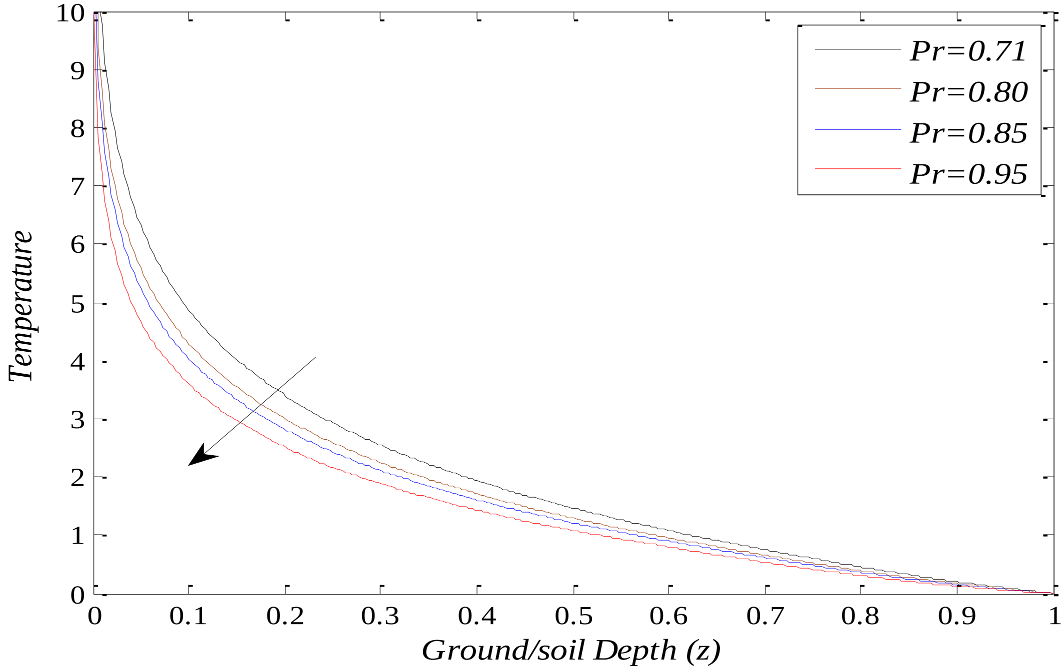

Figure 3: Temperature profile for various values of Prandtl number for **unsaturated** sandy soil

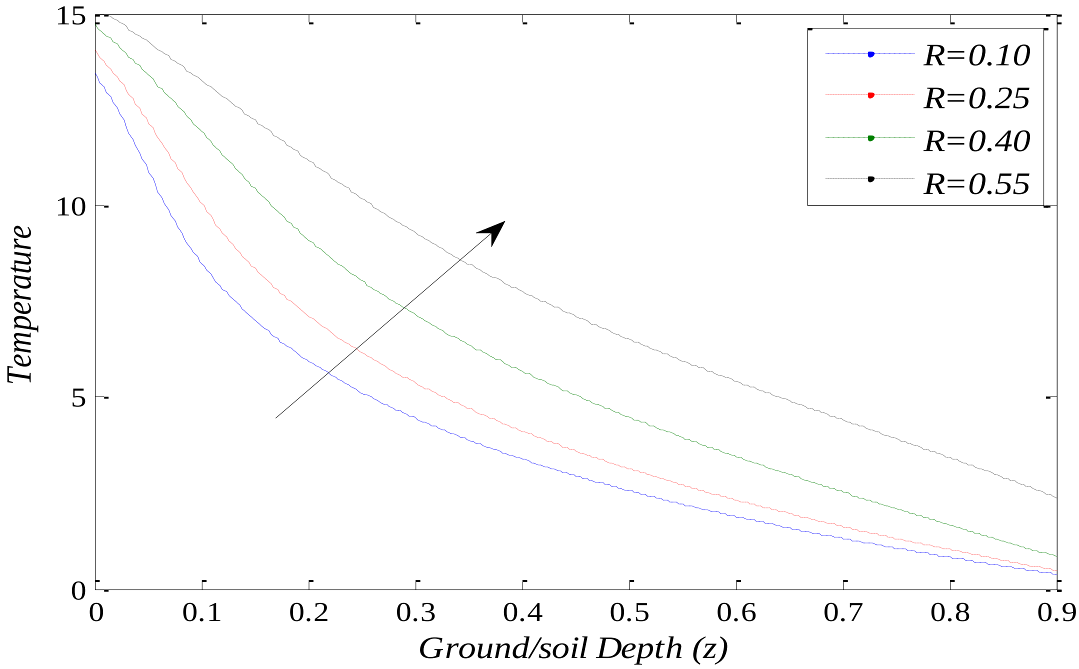

Figure 4: Temperature profile for various values of Radiation parameter for **saturated** sandy soil

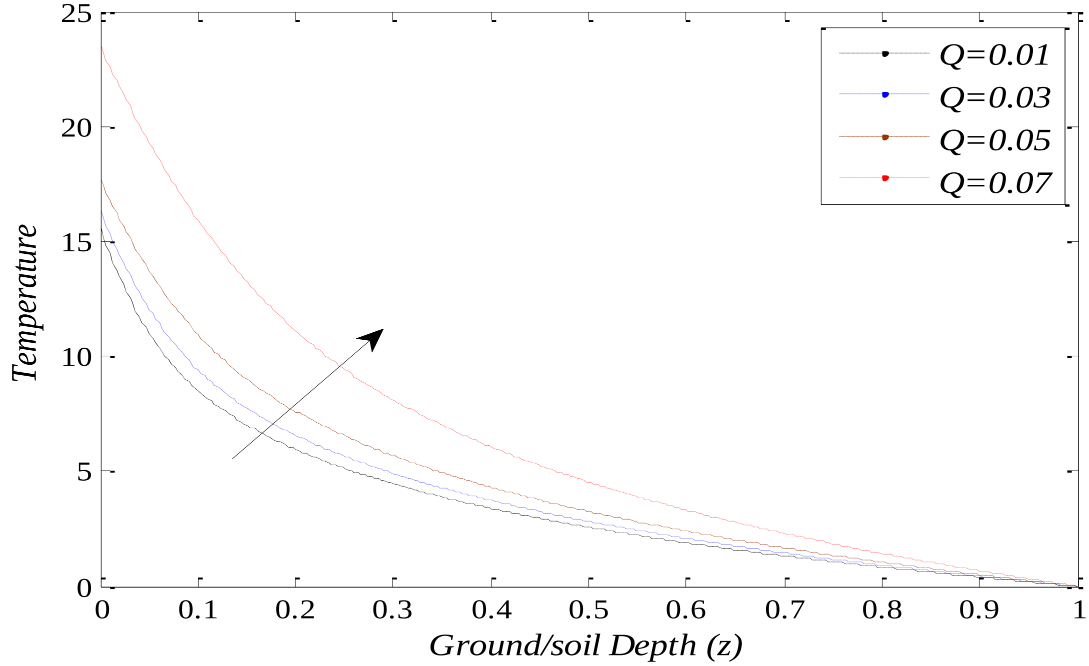

Figure 5: Temperature profile for various values of internal heat generation parameter for **saturated** sandy soil

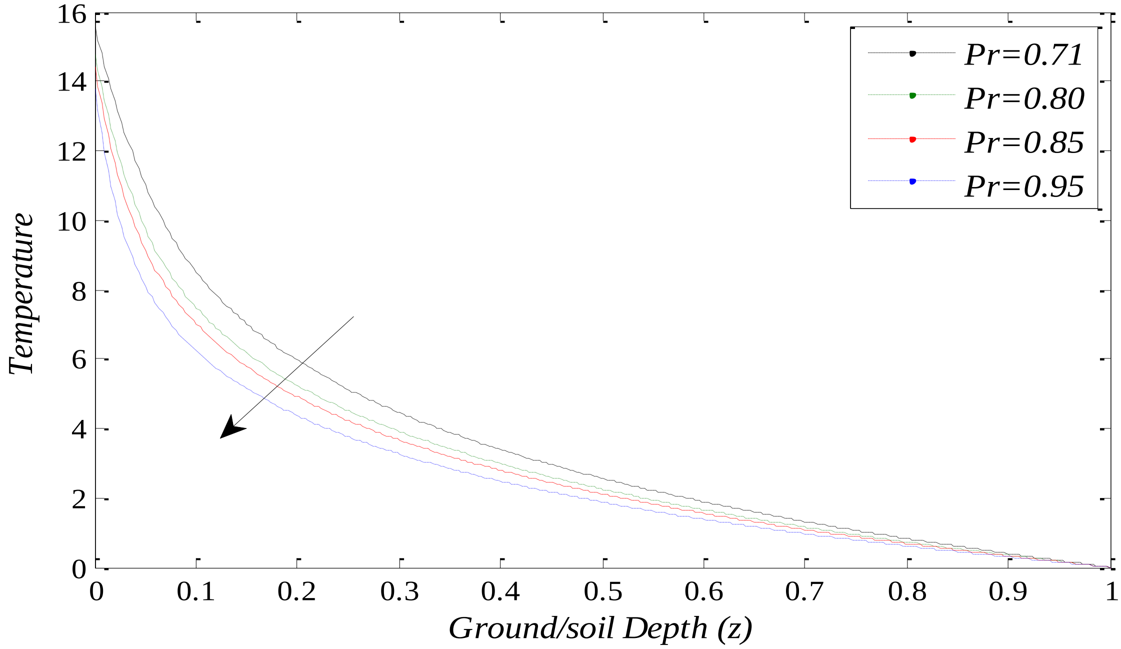

Figure 6: Temperature profile for various values of Prandtl number for **saturated** sandy soil

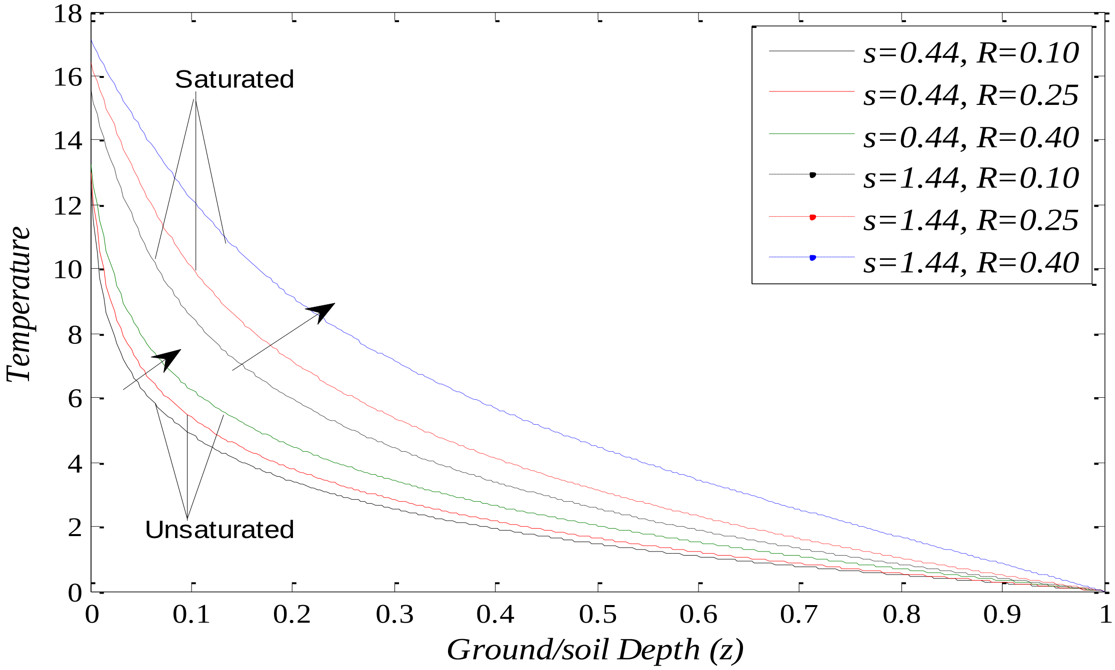

Figure 7: Temperature profile for various values of Radiation parameter for both **unsaturated** and **saturated** sandy soil.

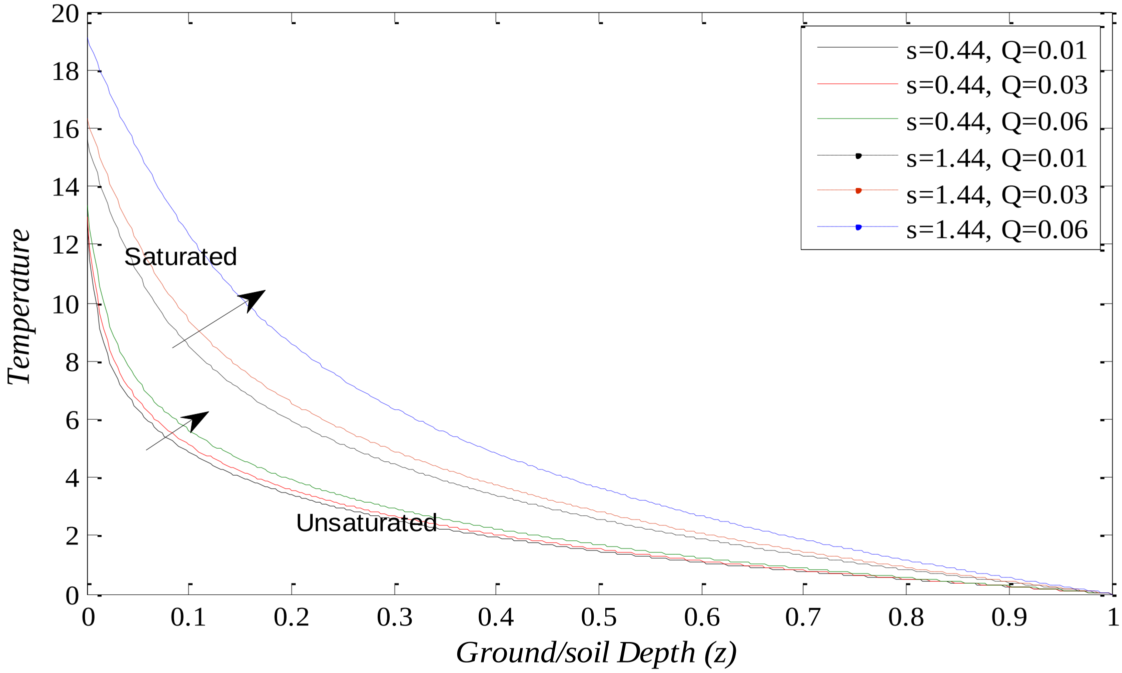

Figure 8: Temperature profile for various values of internal heat generation parameter for both **unsaturated** and **saturated** sandy soil.

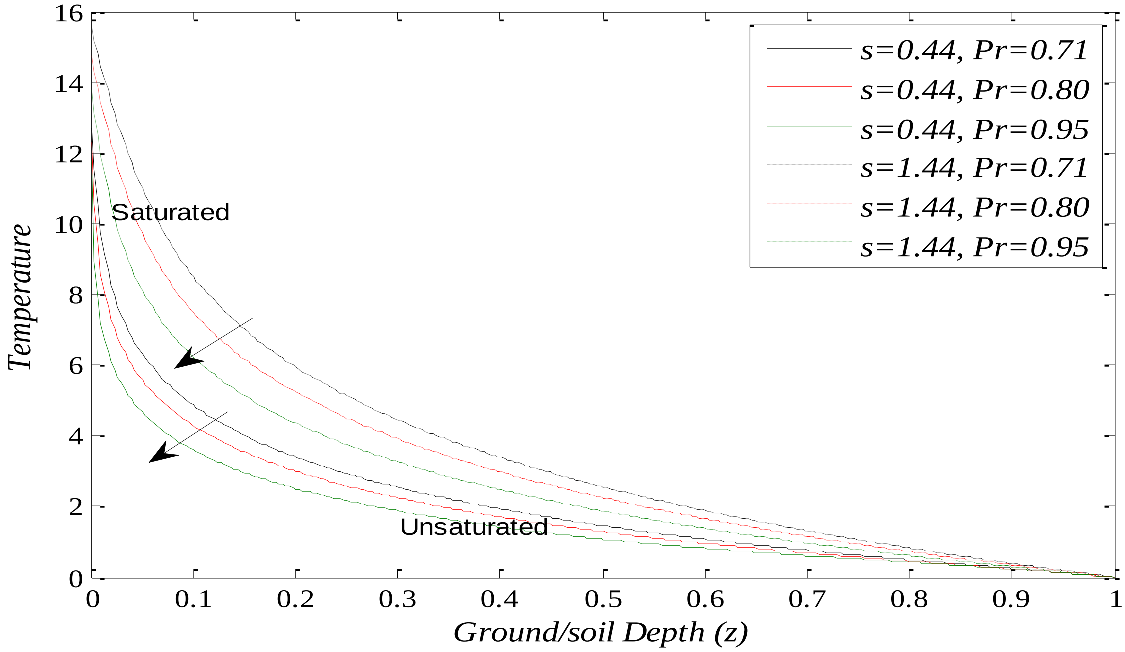

Figure 9: Temperature profile for various values of Prandtl number for both **unsaturated** and **saturated** sandy soil.

Figures 1 and 4 depict the temperature profiles for different values of radiation parameter for unsaturated and saturated sandy soil. The results show that as the solar radiation increases, the temperature for both unsaturated sandy soil and when it is saturated with water increase as well.

Figures 2 and 5 represent the temperature profiles for different values of internal heat generation for both the unsaturated and saturated sandy soil. It is clearly seen that the increasing internal heat also boost the temperature of sandy soil in its unsaturated form and also when it is saturated.

In figures 3 and 6 which are the temperature profiles for various Prandtl number for the unsaturated and saturated sandy soil. As usual, when the Prandtl number increased, the temperature of the soil at the boundary layer in both states of been unsaturated and when it is saturated decreased.

Meanwhile, figures 7, 8 and 9 represent the comparison of effects of increasing solar radiation, the internal heat generation and the Prandtl number on the temperature of unsaturated and saturated sandy soil at increasing depth of the soil. It is observed that though the temperature of the soil increased at both state when the solar radiation and the internal heat increase, yet the level and the rate of these increments differ. It is noticed that on both cases, the level and rate at which the temperature increases when the sand is saturated with water is more than when it is unsaturated. This is because the wet media conduct and transfer heat faster than the dry ones. Also, according to figure 9, the level and the rate at which heat diffuses away from the boundary layer of the saturated sand is more than that of the unsaturated sand.

5. Conclusion

The effects of solar radiation and internal heat generation on both unsaturated and saturated sandy soils with variable suction velocity and time-dependent thermal conductivity have been considered and the following conclusions were made. The mounting solar radiation and internal heat from the earth's crust increased the temperature of the sandy soil both when unsaturated and when saturated with water. The saturated sand has the higher temperature with greater rate of increase when both physical factors are been boosted. Nonetheless, the temperature converged to a mean earth temperature of about 30 to 50 feet into the ground which is a dimensionless value 1 on the graph.

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Appendix

$$m_1 = -\frac{P_r}{2(1+st)} + \sqrt{\frac{P_r^2}{4(1+st)^2} - \frac{P_r Q}{1+st}}$$

$$m_2 = -\left(\frac{P_r}{2(1+st)} + \sqrt{\frac{P_r^2}{4(1+st)^2} - \frac{P_r Q}{1+st}}\right)$$

$$m_3 = -\frac{P_r}{2(1+st)} + \sqrt{\frac{P_r^2}{4(1+st)^2} - \frac{P_r Q}{1+st} + \frac{P_r i\omega}{1+st}}$$

$$m_4 = -\left(\frac{P_r}{2(1+st)} + \sqrt{\frac{P_r^2}{4(1+st)^2} - \frac{P_r Q}{1+st} + \frac{P_r i\omega}{1+st}}\right)$$

$$C_1 = -C_3 e^{-m_1 z} \qquad C_2 = 1 + C_3\left(e^{-m_1 z} - 1\right) \qquad C_3 = R^2 / Q$$

$$C_4 = \frac{-C_6 e^{m_1 z}}{e^{m_3 z}} \qquad C_5 = -(C_4 + C_6 + C_7)$$

$$C_6 = \frac{-P_r A m_1 C_1}{m_1^2 + stm_1^2 + P_r m_1 + P_r Q - P_r i\omega}$$

$$C_7 = \frac{-P_r A m_2 C_2}{m_2^2 + stm_2^2 + P_r m_2 + P_r Q - P_r i\omega}$$